

Angle Invariance for Distance Measurements Using a Single Camera

J. C. Aparício Fernandes *

J. A. B. Campos Neves **

* Dept. Industrial Electronics , Universidade do Minho, Guimarães, Portugal

** Dept. Electrical Engineering, Universidade Lusíada, Famalicão, Portugal

* aparicio@dei.uminho.pt

** jacn@netcabo.pt

Abstract - Various types of sensors are available to implement distance measurement for mobile robots. Scene recognition and path planning point to the use of optical imaging systems and machine vision approaches. For middle size robots, such as those used in robotic football league, reduced weight and volume are mandatory, and a single camera fixed on the robot is the usual choice. 3D localization of objects with such a simple system is impossible, unless some knowledge of the environment and/or objects is available. Localization in 3D space needs three coordinates. The common central projection used in linear image produces a 2D image, from which only two coordinates can be extracted. For the central projection system, any point in a straight line to the lens optical centre of the lens has the same image pixel representation. The distance from the object to the optical centre is the unknown coordinate to be obtained by processing other information. In the case of mobile robots, the movement is usually on a plane surface, meaning that height and camera orientation remain constant. If the object is also at a fixed height, its possible positions define a horizontal plane, and its particular location can be obtained by intersecting this plane with the straight line defined by the corresponding image point on the sensor and the lens optical centre. This work presents both a calculation method and also a calibration procedure for this setup.

This paper concerns the use of linear optical systems, where angles to the optical axis are maintained for both image and object sides. For optical systems with radial distortions, such as barrel and pincushion types, a one-dimensional function relating object and image side angles can be used to compensate the non linearity and allow this approach to be used.

Also in vision systems, where mirrors are combined with normal lens to allow 360° viewing, similar image-object angular relationships can be used to compute target positions.

I. INTRODUCTION

Mobile robots, such as those used in middle size robotic football league, as shown in Fig. 1, move on a plane surface carrying a single fixed camera. Fig. 2 is an example of the images obtained with this type of system using the usual linear (or quasi-linear) system.

Walls and the goal-keeper robot are the obstacles in this example and the free space in front allows the possible movements to be planned. A target can also be defined; for instance, it could be defined as the right edge of the goal area, marked *T* on the image. How far is it and at what direction?

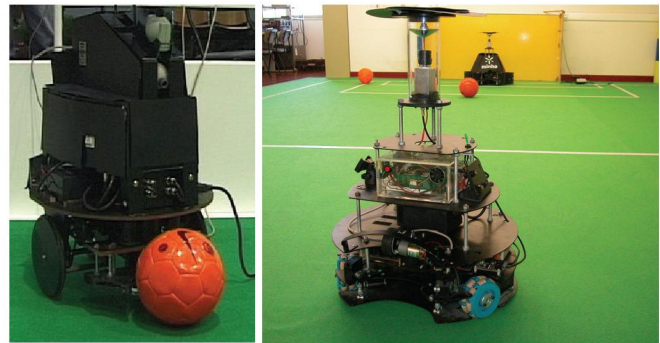


Fig. 1. Examples of mobile robots for middle size football league

This is the question this approach gives an answer for, using a single conventional camera and knowledge about the geometric characteristics of the environment. For instance, in the case of Fig. 2, the target *T* is on the floor. This is used to simplify the calculation process, as described in the following sections. Other cases, as the red ball in Fig. 1, its centre is at known height - half diameter of the ball. For these cases, the general 3D location problem becomes a 2D situation, that can be solved using just one image.



Fig. 2. Example of image with localization of optical centre *P* and target *T*

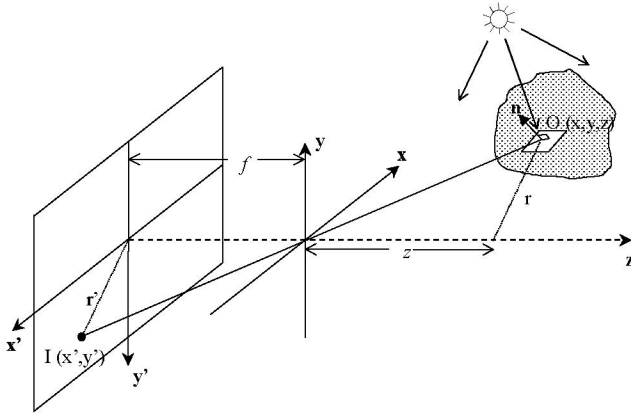


Fig. 3. The image formation system

II. THE OPTICAL SETUP

The standard optical system provides an image on the planar sensor with the geometry sketched in Fig. 3 [1, 2].

Once obtained a focused image for an object at a distance, between the closest and the furthest positions of interest, this distance adjustment can remain fixed, the depth of field covering for the other object distances. This means that the distance between the optical central of the lens and the image plane remains constant throughout the process. It is marked as f on the diagram in Fig. 4, for it is approximately the lens focal distance or, when adjusting the focus for a far object, it really corresponds to it. The lens results can be modeled by the simple central projection model. The optical centre is usually chosen as the origin of the xyz system, as in Fig. 3.

For the mobile robot, the setup can be sketched as in Fig. 4.

The camera is fixed on top for the best viewing position and tilted downwards, pointing to P . The optical axis $O'P$ makes an angle δ to the horizontal. The point O is the vertical projection on the floor of O' , the optical centre of the lens, and its height h remains constant while the robot moves around.

Considering an identifiable point of the image, whose height

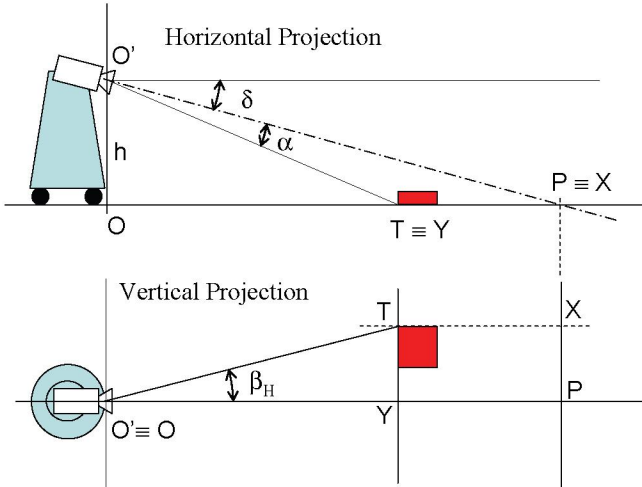


Fig. 4. Horizontal and vertical projections of the setup with the definitions of the variables

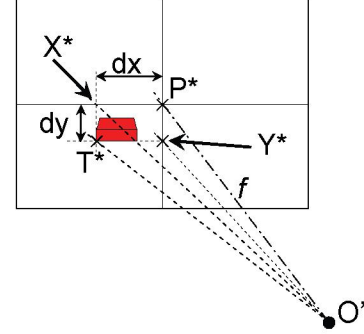


Fig. 5. Image formation diagram, with * for corresponding object points in Fig. 4

is known, specially those points known to be on the floor, as the case of the target point T in the sketches of Fig. 4 and image of Fig. 2 and using measured values for both the height h of the optical centre and the angle of tilt δ , some relations can be established:

$$\frac{h}{OP} = \tan \delta \quad (1)$$

$$\frac{h}{OX} = \tan (\delta + \alpha) \quad (2)$$

$$\frac{TX}{OX} = \tan \beta \quad (3)$$

$$OT = \frac{OX}{\cos \beta} \quad (4)$$

$$\frac{h}{OX} = \tan (\delta + \alpha) \quad (5)$$

Knowing h and OP and using (1) and (2) the distance OX depends on knowing α . If also known β , the location of the target can be computed.

III. THE ANGLE INVARIANCE

The central projection, detailed in Figs. 3 and 4, maintains the corresponding object and image angles (noted * for the corresponding image positions):

$$\alpha = \angle P^* O' Y^* = \angle PO' Y \quad (5)$$

and

$$\beta = \angle P^* O' Y^* = \angle PO' Y. \quad (6)$$

Adjusting the camera position to make the xx' image axis horizontal is the normal practice, that assures no horizontal tilt and it also means that P^*X^* is parallel to XX' axis. The optical axis being normal to image plane, angles α and β can be calculated from image data once known the distance O^*P^* (referred as f in the setup description):

$$\alpha = \arctan\left(\frac{dy}{f}\right) \quad (7)$$

and

$$\beta = \arctan\left(\frac{dx}{f}\right). \quad (8)$$

When lens distortion are to be accounted for, correction procedures can be used to restore the angle invariance, correcting the image angles in (7) and (8) for the object angles from the angle transfer function, as proposed in [3].

IV. CALIBRATION

The quality of the results is dependent of the constants used, namely f , δ , OP and also the coordinates of the optical centre of the image, most often not identical to the geometric centre.

The first step is the determination of the optical centre coordinates of the camera system, ideally in the centre of the image. It can be found around it when there is a slight misalignment of the sensor with the lens mount. This must be done before fixing it on the robot.

A simple method is to rotate the camera around the lens axis. The typical cylindrical construction of the lens facilitates this procedure. The fixed point of the images is the optical image centre and its (x_0, y_0) coordinates are to be retained. For cameras with interchangeable lens, this procedure can be done for just one of the available lenses and kept for the all the set.

The next step is to fix the camera to the robot and adjusting its position for no lateral tilt and just only the desired vertical angle. For this, two orthogonal straight lines are to be drawn on the floor. The robot is positioned above one and the crossing adjusted to match the optical image centre. The horizontal tilt is adjusted till the crossing line appears horizontal on the image. At this point, the camera position can be locked.

Two measurements are needed: the height of the optical centre and its horizontal distance to the crossing point. This correspond, respectively, to $h=OO'$ and OP on the diagrams of Fig. 4. The optical centre can usually be considered as corresponding to the centre of the external element of the lens. The tilt angle can be computed using (1).

A different method can be used to get this vertical tilt: all points at the same height as the lens optical centre are located on the image on a straight line, positioned horizontally if there is no lateral tilt of the camera. With just two of these points, the lateral tilt of the camera can be measured and accounted for or simply used to find the camera position with no lateral tilt. The distance of the optical image centre to this same height

point is measured in terms of image coordinates. Using the lens focal distance, the tilt angle can be computed using (10). This procedure needs the pixel size values and also the precise focal distance of the lens (that can be different of the nominal value usually inscribed on the lens mount). Combined with the previous method or any other image size calibration technique, this procedure allows the computation of the lens focal distance, either in pixel size units – sufficient for the envisaged application – or in SI distance units, once the pixel size is known.

From the external measurements, the tilt angle δ is, from Fig. 4,

$$\delta = \arctan\left(\frac{h}{OP}\right) \quad (9)$$

but also, and in relation to Fig. 5,

$$\delta = \arctan\left(\frac{dy}{f}\right) \quad (10)$$

$$f = dy \cdot \frac{h}{OP}. \quad (11)$$

This equal angle condition, for both object and image, is the core of this work: from the image side, the angles can be easily computed and its value used to compute the distances for the object side.

V. CALCULATION PROCEDURE

After obtaining the pixel coordinates (x, y) from the image processing algorithm of the point of interest, and with the location coordinates of the optical centre of the image (x_0, y_0) the relative coordinates (dx, dy) are computed

$$dx = x - x_0 ; dy = y_0 - y, \quad (12)$$

inverting the signal of the vertical coordinate, to put it in the normal cartesian reference, where positive dy values correspond to points above de xx' axis.

The angle α is calculated by

$$\alpha = \arctan\left(\frac{dy}{f}\right) \quad (13)$$

and the horizontal distance OY :

$$OY = \frac{h}{\tan(\delta - \alpha)} \quad (14)$$

where δ is the constant calculated in (10).

$$\tan \beta = \frac{dx}{f} \quad (15)$$

Considering β the angle on a plane that contains the optical axis and whose projection on the horizontal plane is β_H , both projections (Fig. 4) coincide, although β_H is greater than β .

To calculate the distance TX we need to project $O'Y$ on the optical axis OP to have d , the distance to the target measured on the optical axis:

$$d = O'Y \cdot \cos \alpha = \frac{OY \cdot \cos(\alpha)}{\cos(\delta - \alpha)} \quad (16)$$

with OY from (14). With distance d and angle β , YT can be obtained by:

$$YT = \frac{dx}{f} \cdot d \quad (17)$$

The angle β_H – the horizontal angular position of the target relative to the mobile robot – is

$$\beta_H = \tan^{-1} \left(\frac{YT}{OT} \right) \quad (18)$$

And in this way it is possible to calculate distance and orientation for different targets just from a single image if the targets are located on the floor. For targets whose height is known, the procedure is the same, but substituting in (9) the height of the optical centre for its difference to the target height.

VI. EXAMPLE

Using the image in Fig. 2, the image coordinates of the indicated points are:

$$\begin{aligned} \text{Image centre X : } x_0 &= 1280, y_0 = 960 \\ \text{Target point T : } x_1 &= 1830, y_1 = 1165. \end{aligned}$$

Also from the image, the y coordinate of points at same height as the optical centre, such as top surface of the mirror of the robot in the image:

$$y_H = 663.$$

From the setup parameters

$$\text{Optical centre height : } h = 750 \text{ mm.}$$

From the camera CCD sensor specifications:

$$\text{Square pixel size : } 3.44 \text{ } \mu\text{m.}$$

From an independent calibration procedure to obtain the correct lens focal distance:

$$\text{Focal distance in pixel size units: } 2065$$

The measured focal lens distance was 7.12, mm for a nominal value of 7.1 mm.

Combining the image coordinate values with the setup parameters, the following values were obtained:

Vertical tilt angle of camera:

$$\delta = \tan^{-1} \left(\frac{y_o - y_H}{f} \right) = \tan^{-1} \left(\frac{960 - 663}{2065} \right) = 8.18^\circ \quad (19)$$

Vertical angle of target point to optical axis:

$$\alpha = \tan^{-1} \left(\frac{y_o - y_1}{f} \right) = \tan^{-1} \left(\frac{960 - 1165}{2065} \right) = -5.67^\circ \quad (20)$$

Horizontal angle of target point to optical axis:

$$\beta = \tan^{-1} \left(\frac{x - x_o}{f} \right) = \tan^{-1} \left(\frac{1830 - 1280}{2065} \right) = 14.9^\circ \quad (21)$$

Distance of target:

$$O'Y = \frac{h}{\tan(\delta - \alpha)} = \frac{750 \text{ mm}}{\tan(8.18^\circ + 5.39^\circ)} = 3107 \text{ mm} \quad (22)$$

$$\begin{aligned} YT &= \frac{OT \cdot \cos(\alpha) \cdot \tan(\beta)}{\cos(\delta - \alpha)} = \\ &= \frac{3107 \text{ mm} \cdot \cos(-5.39^\circ) \cdot \tan(14.9^\circ)}{\cos(8.18^\circ + 5.39^\circ)} = 846 \text{ mm} \end{aligned} \quad (23)$$

Horizontal angular position of the target

$$\beta_H = \tan^{-1} \left(\frac{YT}{OT} \right) = \tan^{-1} \left(\frac{846}{3107} \right) = 15.23^\circ \quad (24)$$

The calculated values corresponded to the measured values within a less than 10% tolerance, which was close enough to plan the objectives of the mobile robot.

VII CONCLUSIONS

The presented procedure allows the prediction of target positions relative to the mobile robot system using only data extracted from a single image and parameters from the setup. It depends on knowing the height of target point above the floor. It just needs to be subtracted from the camera height h to use the described procedure to locate the target.

For catadioptric systems, using mirrors to allow 360° panorama views in single images (the robot in Fig. 2 uses a conical mirror), the angle invariance principle is kept and the same basis can be adapted for solving target locations, as described in another report [4].

We plan to test the viability of this method to measure speeds of cars using the standard traffic surveillance camera systems.

ACKNOWLEDGMENTS

This work has been developed at the Algoritmi Center, University of Minho, sponsored by Fundação para a Ciência e Tecnologia (FCT), Portugal.

REFERENCES

- [1] E. Hecht, A. Zajac, *Optics*, Addison-Wesley, NJ: 1974.
- [2] Fernandes, J.C.A., *Análise de Imagens Texturadas com base em Matrizes de Co-ocorrência Modificadas*, 2004, Universidade do Minho, Guimarães.
- [3] J. C. A. Fernandes, M. J. O. Ferreira, J. A. B. C. Neves, and C. A. C. Couto, "Fast Correction of Lens Distortion for Image Applications," presented at IEEE International Symposium on Industrial Electronics, ISIE'97, Guimaraes, Portugal, 1997.
- [4] Fernandes, J. C. A., Neves, J. A. B. C., "Using Conical and Spherical Mirrors with Conventional Cameras for 360° Panorama Views in a Single Image," accepted for ICM 2006 - IEEE International Conference on Mechatronics, July 3-5, 2006, Budapest, Hungary.