The problem of finding and tracking the mobile robot's pose is affected mainly by the robot configuration and the parameters characterizing the systematic odometry errors.

Within the significant toolbox of mathematical tools that can be used for stochastic estimation from noisy sensor measurements, one of the most well known and often used tools is the Kalman filter. As an extension to the same idea, the extended Kalman filter (EKF) is used if the dynamic of the system and/or the output dynamic is nonlinear. It is based on linearization about the current estimation error mean and covariance [11].

This paper presents a new design of augmented extended Kalman filter for real-time simulation of mobile robots using Simulink®. We design the augmented extended Kalman Filter to fuse the absolute measurement's data and the odometry from the wheels' encoders for real-time reconstruction of mobile robots position and orientation with an on-line odometry calibration.

The remainder of this paper is organized as follows. First, we present the robot model for a two-wheeled differential drive mobile robot. Then, the AKF is introduced. In the next section, we describe the algorithmic details of the EKF formulations and implement the augmented version of this filter for the differential drive mobile robots. Next, the Simulink implementation of the system is presented. The following section shows the simulation results and discusses their significance. And a final section concludes the paper including some hints for future study.

1. INTRODUCTION

The problem of finding and tracking the mobile robot's pose is called localization, and can be global or local. During the past few years many suggestions have been made to address this problem. One of the first methods introduced, and still used in many projects, is odometry. It is the reconstruction of the mobile robot configuration, i.e., position and orientation, resorting to the encoders readings, and assuming an absolute measurement available, the AEKF provides the local reconstruction of mobile robots position and orientation with an on-line odometry calibration. The simulation results verify the effectiveness of the proposed method and suggest it as a promising way for real time implementations of augmented kalman filters.

Index Terms --- Localization, Odometry calibration, Encoder, augmented, extended Kalman filter, real-time

2. ROBOT MODEL

A two-wheeled differential drive mobile robot operating on a planar surface is sketched in Figure 1. Where \( x \) and \( y \) denote the position of the center of the axle of the robot with respect to the global coordinate frame, \( \theta \) denotes the heading of the vehicle with respect to the \( x \) axis in the global coordinate frame. There are two main wheels, each of which is attached to its own motor. A third wheel – a caster, is placed in the front to passively roll along while preventing the robot from falling over.

Deriving a model for the whole robot's motion is a bottom-up process. Each individual wheel contributes to the robot's motion and, at the same time, imposes constraints on robot motion. Kinematics refers to the evolution of the position and velocity of a mechanical system, without reference to its mass and inertia. The differential equations that describe the kinematics of the robot are written as

\[
\begin{align*}
\dot{x} &= v \cos \theta \\
\dot{y} &= v \sin \theta \\
\dot{\theta} &= \omega
\end{align*}
\]  

(1)
where the absolute velocity of the robot body \( v \) and the angular speed \( \omega \) are assumed to be inputs.

The robot has independent velocity control of each wheel. The motion of the left and right wheels are characterized by angular velocities \( \omega_R \) and \( \omega_L \), respectively. The body-fixed components \( v \) and \( \omega \) of the robot velocity are related to these angular velocities by

\[
v = \frac{r_R}{2} \omega_R + \frac{r_L}{2} \omega_L
\]

\[
\omega = \frac{r_L}{d} \omega_R - \frac{r_R}{d} \omega_L
\]

in which \( d \) denotes the distance between the two wheels, while \( r_R \) and \( r_L \) are the radii of the right and left wheels, respectively [1].

The main feature of this model for wheeled mobile robots is the presence of non-holonomic constraints, due to the rolling without slipping condition between the wheels and the ground. The non-holonomic constraints impose that the generalized system velocities, i.e., \( \dot{x}, \dot{y} \) and \( \dot{\theta} \), cannot assume independent values. It can be observed that the kinematic model in equation 1 does not include the dynamic effects of the robot body and actuators.

3. AUGMENTED KALMAN FILTER

Let us define \( \vec{x} \) as the robot configuration

\[
\vec{x} = [x, y, \theta, r_L, r_R, d]^T
\]

(3)

The distance between the wheels or the wheel radii may not be precisely known, which lead to systematic errors and worsen the performance of the Kalman filter. Making a complex model of the two-wheeled differential drive mobile robots will not reduce this problem but in fact make it worse, as this model not only will contain the same errors, but most likely also introduce further inaccuracies [6].

To remedy this problem, we adopt the same augmented kalman filter (AKF) introduced by Martinelli [8] and Larsen [6]. This filter estimates the augmented state containing the robot configuration and the systematic parameters as

\[
\vec{x}_a = [x, y, \theta, r_L, r_R, d, \delta]^T
\]

(4)

In the following chapter, we implement the so-called augmented extended kalman filter to simultaneously localize the vehicle and estimate the vehicle odometry.

4. EXTENDED KALMAN FILTER

The Kalman filter addresses the general problem of trying to estimate the state \( x \in \mathbb{R}^n \) of a discrete-time controlled process that is governed by a linear stochastic difference equation. As an extension to the same idea, the extended Kalman filter (EKF) is used if the dynamic of the system and/or the output dynamic is nonlinear. EKF is based on linearization about the current estimation error mean and covariance [11].

4.1. Definitions

Let us assume that the process has a state vector \( x \in \mathbb{R}^n \) and a control vector \( u \) and is governed by the non-linear stochastic difference equation

\[
x_k = f(x_{k-1}, u_k, w_{k-1})
\]

with a measurement \( z \in \mathbb{R}^m \) that is

\[
z_k = h(x_k, v_k)
\]

the random variables \( w_k \) and \( v_k \) represent the process and measurement noise, respectively. They are assumed to be independent of each other, white, and with normal probability distributions with covariance matrices \( Q \) and \( R \).

Defining \( \tilde{x}_k \) as the a posteriori estimate of the state (from previous time step \( k \)) one can approximate the state and measurement vector without the noise effects as

\[
\tilde{x}_k = f(\tilde{x}_{k-1}, u_k, 0)
\]

(7)

and

\[
\tilde{z}_k = h(\tilde{x}_k, 0)
\]

(8)

To estimate the non-linear process a first order Taylor expansion is performed

\[
x_k \approx \tilde{x}_k + A(x_{k-1} - \tilde{x}_{k-1}) + Ww_{k-1}
\]

\[
z_k \approx \tilde{z}_k + H(x_k - \tilde{x}_k) + Vv_k
\]

(9)

where \( x_k \) and \( z_k \) are the actual state and measurement vectors, \( \tilde{x}_k \) is an estimate of the state at step \( k \), \( \tilde{z}_k \) are \( w_k \) and \( v_k \) are representing the process and measurement noise and \( \tilde{x}_k \) and \( \tilde{z}_k \) are the approximate state and measurement vectors. It can be shown that the time update equations of EKF is

\[
\tilde{x}_k = f(\tilde{x}_{k-1}, u_k, 0)
\]

(10)

\[
P_{k|x} = A_k P_{k-1|x} A_k^T + W_k Q_k W_k^T
\]

where \( \tilde{x}_k \) is the a priori state estimate [11]. These time update equations project the state and covariance estimate (\( P_k \)) from the previous time step \( k - 1 \) to the current time step \( k \). And the measurement update equations of the EKF are

\[
K_k = P_{k|x} H_k^T (H_k P_{k|x} H_k^T + V_k R_k V_k^T)^{-1}
\]

\[
\tilde{x}_k = \tilde{x}_k + K_k (z_k - h(\tilde{x}_k, 0))
\]

(11)

\[
P_k = (I - K_k H_k) P_{k|x}
\]

where \( A, W, H \) and \( V \) are Jacobian matrices and \( K \) is the correction gain vector. These measurement update equations correct the state and covariance estimate with the measurement \( z_k \). The design process of this filter is explained next.

4.2. Implementation

The discrete model of the system can be derived by converting the differential equation 1 to their corresponding backward difference equations. So, the dynamical equation for the augmented state \( \vec{x}_a \) is given by:
5. SIMULINK IMPLEMENTATION

The algorithm has been implemented and simulated in Simulink®. The block diagram used in simulation of the AEKF for the differential drive mobile robot is depicted in Figure 2. Figures 3-5 show the subsystems illustrated in Figure 2. The kinematics equations of the robot are inserted into the Simulink using an S-Function block, i.e. “robot”. Also, a MATLAB Fcn block is added to perform the AEKF algorithm; and a zero-order hold (ZOH) is inserted into the filter algorithm to model the digital to analog conversion.

\[
\begin{bmatrix}
x \\
y \\
\theta \\
\tau_l \\
\tau_r \\
d \end{bmatrix}_{k+1} =
\begin{bmatrix}
x \\
y \\
\theta \\
\tau_l \\
\tau_r \\
d \end{bmatrix}_k +
\begin{bmatrix}
x \\
y \\
\theta \\
\tau_l \\
\tau_r \\
d \end{bmatrix}_k
\]

where the subscript \( k \) denotes the k-th time sample and \( T \) is the sampling period.

The Jacobian matrix at each iteration can be derived as

\[
A =
\begin{bmatrix}
1 & 0 & A_{13} & A_{14} & A_{15} & 0 \\
0 & 1 & A_{23} & A_{24} & A_{25} & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

With

\[
A_{13} = \frac{\partial f_1}{\partial \theta} = - \frac{1}{2} \left( r_l \omega + r_r \omega \right) \sin \theta
\]

\[
A_{14} = \frac{\partial f_1}{\partial \tau_l} = \frac{1}{2} \left( \tau \omega \right) \cos \theta
\]

\[
A_{15} = \frac{\partial f_1}{\partial \tau_r} = \frac{1}{2} \left( \tau \omega \right) \cos \theta
\]

\[
A_{23} = \frac{\partial f_2}{\partial \theta} = \frac{1}{2} \left( r_l \omega + r_r \omega \right) \cos \theta
\]

\[
A_{24} = \frac{\partial f_2}{\partial \tau_l} = \frac{1}{2} \left( \tau \omega \right) \sin \theta
\]

\[
A_{25} = \frac{\partial f_2}{\partial \tau_r} = \frac{1}{2} \left( \tau \omega \right) \sin \theta
\]

\[
A_{34} = \frac{\partial f_3}{\partial \tau_l} = - \frac{\tau \omega}{d}
\]

\[
A_{35} = \frac{\partial f_3}{\partial \tau_r} = \frac{\tau \omega}{d}
\]

\[
A_{36} = \frac{\partial f_3}{\partial \theta} = \frac{\tau \omega}{d} - \frac{r_l \omega - r_r \omega}{d^2}
\]

Suppose that the position and orientation of the robot are measured directly. Hence

\[
\bar{Z}_k = H \bar{x}_k + V_k
\]

Where \( V_k \) represents the measurement noise and the output matrix \( H \) is constant and defined as

\[
H = \begin{bmatrix} I_3 & 0_3 \end{bmatrix}
\]

where \( I_3 \) is the \( 3 \times 3 \) identity matrix and \( 0_3 \) is a \( 3 \times 3 \) matrix.

Due to the recursive nature of the EKF algorithm, the state vector needs to be initialized in startup. The initial robot configuration is taken as the first measured value. Also, the nominal values of \( d, \tau_l \), and \( \tau_r \) are chosen as their initial estimations. Here, the following initial conditions are selected randomly for the state vector:

\[
\bar{x}_{\text{initial}} = \begin{bmatrix} 1.5 & 1.25 & 0 & 0.8 & 0.8 & 5.333 \end{bmatrix}^T
\]

We add uncertainty to the initial condition by selecting

\[
P_0 = I_6
\]

and we choose the process noise and measurement noise as

\[
Q = \text{diag} \begin{bmatrix} 0.01, 0.02 \end{bmatrix}
\]

\[
R = \text{diag} \begin{bmatrix} 0.05, 0.05 \end{bmatrix}
\]

Thus we developed all the necessary elements of the augmented EKF for the two-wheeled differential drive mobile robot.

ISMA09-3
6. SIMULATION RESULTS

This section presents simulation results by Simulink®. Figure 6 shows the trajectory estimation of AEKF for the mobile robot. To evaluate the performance of AEKF, we plotted the estimated and actual states for the available measurements in Figures 7-9. Also, Figure 10 shows the estimation error of AEKF. These figures verify the performance of the designed AEKF using Simulink.

In addition, Figures 11-13 illustrate the values of estimated parameters via the augmented filter.
7. CONCLUSIONS

In this research, a new design of augmented extended kalman filter for real-time simulation of mobile robots is presented. A Simulink® model is developed to localize mobile robots while estimating a proper set of odometric parameters. The simulation results verify the effectiveness of the new design. In the future we will conduct some experiments using the Khepera II mobile robot, manufactured by K-Team, to verify the simulations of this research. Furthermore, utilizing the AEKF in real time manner for the simultaneous localization and mapping (SLAM) problem would be our future challenging task, where on-line odometry calibration could play a very important role especially in solving the data association problem.

8. REFERENCES


